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Graphene and Unconventional Supersymmetry

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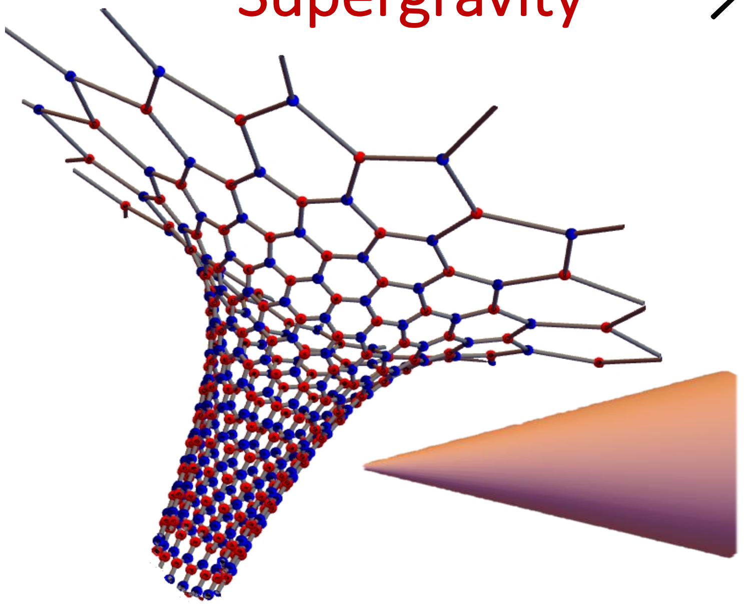
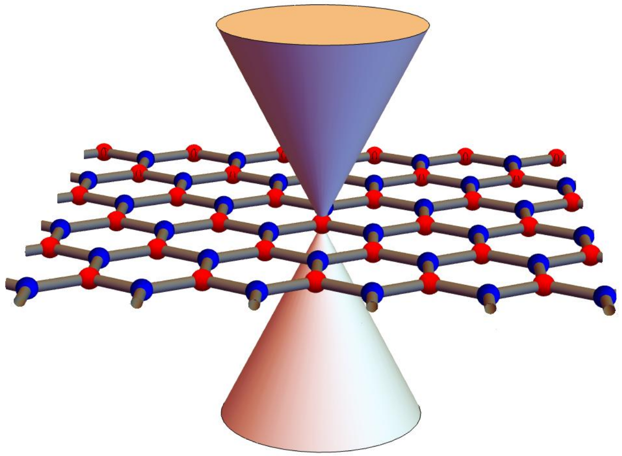
based on L. Andrianopoli, BLC, R. D'Auria, M. Trigiante, JHEP04(2018)007, arXiv:1801.08081;
L. Andrianopoli, BLC, R. D'Auria, A. Gallerati, R. Noris, M. Trigiante, J. Zanelli, JHEP01(2020)084, arXiv:1910.03508;
L. Andrianopoli, BLC, P.A. Grassi, M. Trigiante, JHEP 1906 (2019) 036, arXiv:1903.04431;
L. Andrianopoli, BLC, R. Matrecano, O. Mišković, R. Noris, R. Olea, L. Ravera, M. Trigiante, JHEP 02 (2021) 141, arXiv:2010.02119;
L. Andrianopoli, BLC, R. Matrecano, R. Noris, L. Ravera, M. Trigiante, Fortsch.Phys. 69 (2021) 10, 2100111, arXiv:2107.10361;
L. Andrianopoli, BLC, R. D'Auria, A. Gallerati, R. Noris, L. Ravera, M. Trigiante, J. Zanelli, in progress

Supergravity → Graphene





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Main idea

Objective: Application of the duality symmetries of supergravity in high energy physics to the study of graphene-like 2D materials in condensed matter.

- **The gauge/gravity correspondence relates a strongly coupled gauge theory to a weakly coupled classical gravity theory in one dimension higher** [J.M. Maldacena, ATMP 2 (1998) 231].

bulk **boundary**

3+1-dim. Supergravity on a curved (anti-de Sitter) spacetime $\xrightarrow{\text{Holographic map}}$ **Electronic properties of a 2+1-dim. Layer of graphene**

- **Top-down approach: Large amout of supersymmetry makes model more predictive**

Relation of the electronic properties of graphene to deformations of the lattice geometry

- **Relevance of supersymmetry in low-energy physics** $\rightarrow$ **Interdisciplinary approach**

Plan

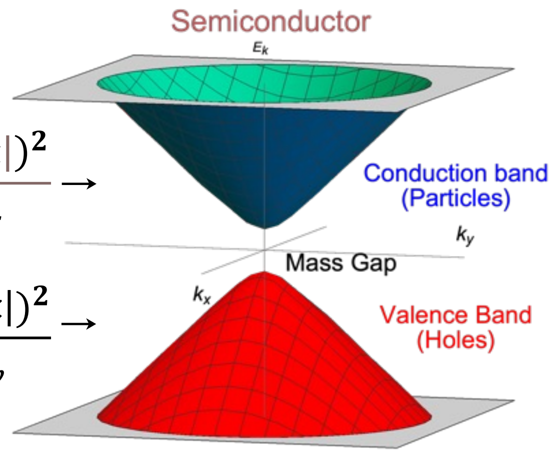
- “Analogue” relativity in condensed matter
  - Geometry in analogue gravity
  - Graphene and the Dirac equation
- The Generalized AVZ Ansatz
  - The Generalized AVZ Model at the boundary of AdS_4 Supergravity
  - The Nieh-Yan-Weyl symmetry
  - Massive Dirac Equation in the AVZ model
- Application to Graphene and the K and K’ valleys
- Comparison with microscopic models of graphene-like 2D materials
- Conclusions and Outlook:
  - Possible generalizations to other models
  - Holography and topology

"Analogue" relativity in condensed matter

Relativistic energy: $E_K = \pm\sqrt{(\hbar c|\boldsymbol{k}|)^2 + (mc^2)^2}$

If *m* finite $E_K \approx E_c + \dfrac{(\hbar c|\boldsymbol{k}|)^2}{m_c^*} \rightarrow$

$E_K \approx E_v - \dfrac{(\hbar c|\boldsymbol{k}|)^2}{m_v^*} \rightarrow$



bare mass $\quad\quad$ **effective mass**

$m \longrightarrow m^*$

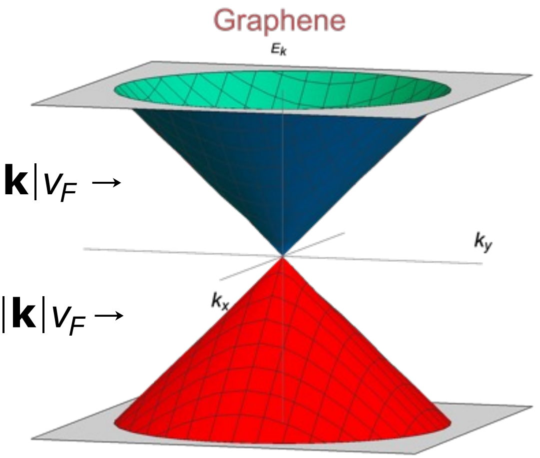
Interaction of the electrons with the lattice atoms $\longrightarrow$ $m^* \leq m$.

In graphene $m^* = 0$.

Massless case:

$E_{\mathbf{k}} = \hbar|\mathbf{k}|v_F \rightarrow$

$E_{\mathbf{k}} = -\hbar|\mathbf{k}|v_F \rightarrow$

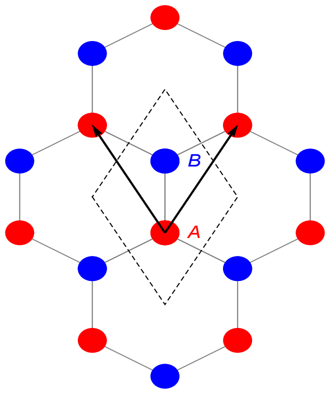


Light speed $\quad\quad$ **Fermi velocity**

$c \longrightarrow v_F = 10^6 \, \frac{m}{s} = \frac{c}{300}$

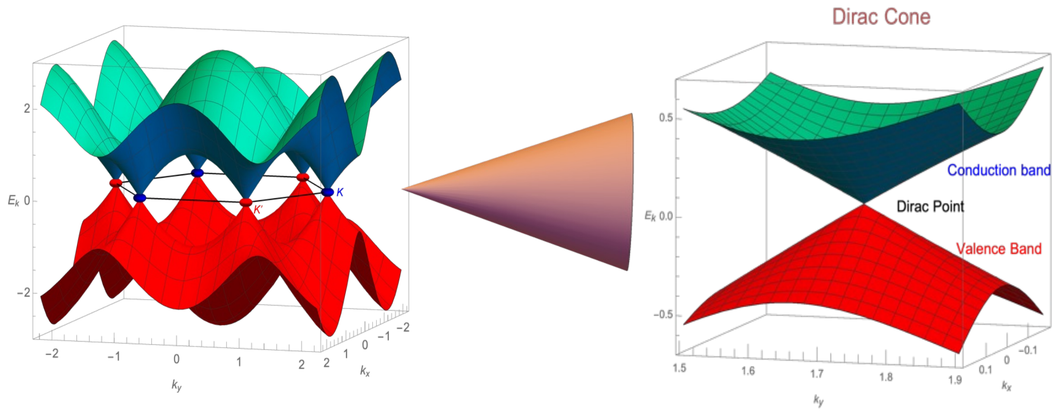
Graphene and the Dirac equation

Two main features of graphene:



Bipartite lattice composed by two triangular sublattices (sites A and sites B):

additional spin-like quantum number: Pseudospin.



At the Dirac points (for a range of 1eV) the spectrum (relation between the energy E_k and the momentum k) is linear:

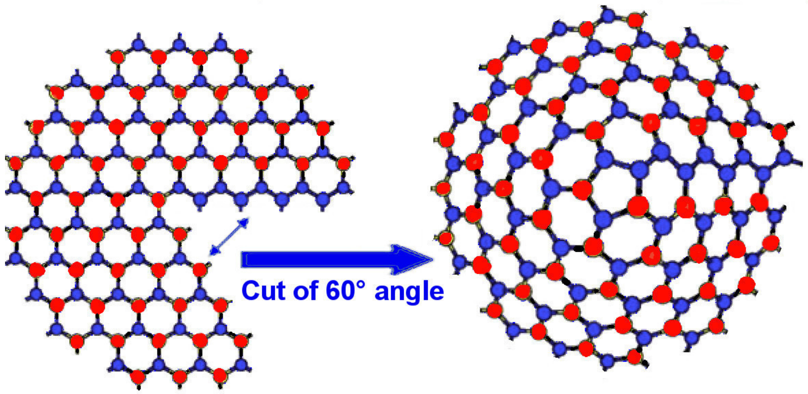
Dirac cone: $E_k = \pm \hbar v_F |k|$

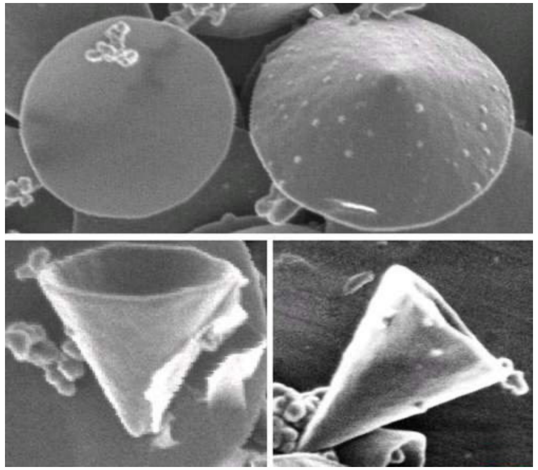
Electrons in graphene obey the same type of equations as a

relativistic Dirac massless particle

Geometry in analogue gravity

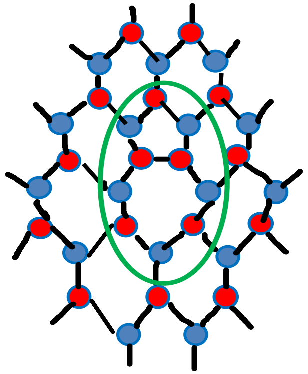
Lattice disclination → **Space Curvature**

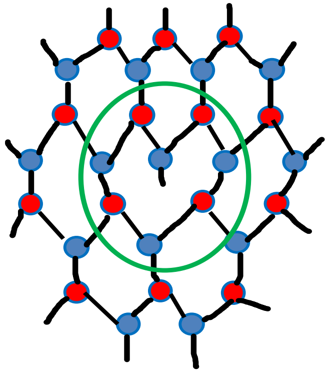




Carbon nanocones
[Kenneth D. Knudsen, https://en.wikipedia.org/wiki/Carbon_nanocone]

Lattice dislocation → **Space Torsion**





Glide and shuffle dislocations in graphene, obtained from subsequent disclinations
[see e.g. A. Carpio, L.L. Bonilla, F. de Juan, M.A.H. Vozmediano, Dislocations in graphene, New J. Phys. 10 (2008)]

The generalized AVZ Ansatz

[L. Andrianopoli, BLC, R. D'Auria, M. Trigiante, *Unconventional Supersymmetry at the boundary of* AdS_4 *Supergravity*, JHEP04(2018)007, arXiv:1801.08081;
L. Andrianopoli, B.L. Cerchiai, R. D'Auria, A. Gallerati, R. Noris, M. Trigiante, J. Zanelli, *N-Extended D=4 Supergravity, Unconventional SUSY and Graphene*, JHEP01(2020)084, arXiv:1910.03508]

In the AVZ model [P.D. Alvarez, M. Valenzuela, J. Zanelli, JHEP 1204 (2012) 058, arXiv:1109.3944] **the fermionic gauge field** ψ_A **is a composite field, and the propagating fermion** χ_A **originates from the radial component of the gravitino** Ψ^A **through the Ansatz**: (A=1,..,$\mathcal{N}$, $\mathcal{N}$ number of supersymmetries; i=0,1,2)

Supergravity Gravitino

$$\boxed{\Psi^A = i\,\gamma_i\, e^i\, \chi^A}$$

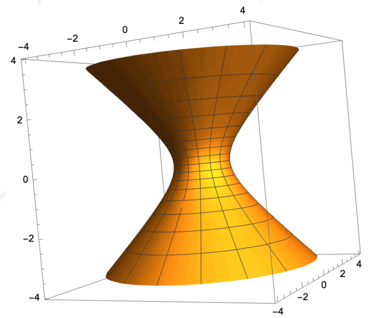
Propagating fermion (pseudoparticle in graphene)

Vielbein Describing the geometry

The matrix γ_i **plays the role of an intertwiner, allowing an identification of the graphene worldvolume** e^i **with the supergravity target space-time** E^i**:**

$$E^i = \mathrm{f}\, e^i$$

The Generalized AVZ Model at the boundary of AdS_4 Supergravity



ADS_4 vacuum of $\mathcal{N}$-extended D=4 supergravity

(super)symmetry: $Osp(\mathcal{N}|4)$

Suitable ultraspinning limit in the Feffermann-Graham parametrization: AdS_3 slicing of AdS_4 (black string) at $r \to \infty$
[A. Gnecchi, K. Hristov, D. Klemm, C. Toldo, O. Vaughan, JHEP 01 (2014) 127, arXiv:1311.1795]

broken to

Achúcarro-Townsend Chern-Simons supergravity on conformal locally AdS_3 boundary located at radial infinity ($r \to \infty$)

(super)symmetry
$OSp(p|2)_+ \times Osp(q|2)_-$;
$p+q=\mathcal{N}$

AVZ Ansatz

AVZ model for propagating spin ½ particle, instrumental in describing the electronic properties of graphene-like 2D materials at the Dirac points

Some Properties of the model

- **There are no bosonic propagating degrees of freedom:**

  Number of bosons $\neq$ Number of fermions

  Unconventional Supersymmetry

- **Supersymmetry is implemented purely as a gauge symmetry**

- **The spin $\frac{3}{2}$ component of the gravitino is projected out, while the spin $\frac{1}{2}$ yields a propagating fermion, suitable to describe pseudoparticles in graphene like 2D materials.**

- **The AVZ Ansatz corresponds to an (unconventional) gauge fixing of supersymmetry in the framework of a BRST quantization** [L. Andrianopoli, B.L. Cerchiai, P.A. Grassi, M. Trigiante, *The Quantum Theory of Chern-Simons Supergravity*, JHEP 1906 (2019) 036, arXiv:1903.04431]

- **The supersymmetry parameter ϵ_A is proportional to the propagating fermion:**

$$\boxed{\epsilon_A \propto (\bar{\chi}\chi)\,\chi_A}$$

The Nieh-Yan-Weyl symmetry

- **The AVZ Ansatz features a local scale invariance, the Nieh-Yan-Weyl (NYW) symmetry** [H. T. Nieh and M. L. Yan, Annals Phys. 138 (1982) 237]:

$$\boxed{e^i \to \lambda(x) e^i, \quad \chi^A \to \frac{1}{\lambda(x)} \chi^A, \quad \lambda(x) \neq 0.}$$

- **It is the breaking of this conformal invariance that turns an originally topological Chern-Simons theory into a system with a propagating spin-1/2 field.**

- **The torsion has a trace part $\boldsymbol{\beta}$ and an antisymmetric part $\boldsymbol{\tau}$:**

$$\mathfrak{D}\, e^i = \boldsymbol{\beta} \wedge e^i + \boldsymbol{\tau}\, \epsilon^{ijk} e_j \wedge e_k$$

- **Under a NYW transformation, one can always set $\boldsymbol{\beta} = 0$ locally.**

- **In a local patch $\overline{\chi_\pm}\chi_\pm$ are constants** ➡

- **The quantities** $\eta_{AB}\overline{\chi}^A\chi^B = \overline{\chi}_+\chi_+ - \overline{\chi}_-\chi_-$, $\overline{\chi}\chi = \overline{\chi}_+\chi_+ + \overline{\chi}_-\chi_-$ **play the role of topological index.**

Difference n_B-n_A between the occupation numbers of graphene electrons in sites A and B

Massive Dirac Equation in the AVZ model

Mass is generated by the **geometric properties of supergravity**, such as torsion, through the action of **supersymmetry**.

From the **Maurer-Cartan equations (consistency conditions)** of the **supersymmetry algebra**

Massive Dirac equation

$$\boxed{\not{\mathfrak{D}}\,\chi = -\frac{3}{2}\,i\,\tau\,\chi}$$

$\mathfrak{D}$ covariant derivative

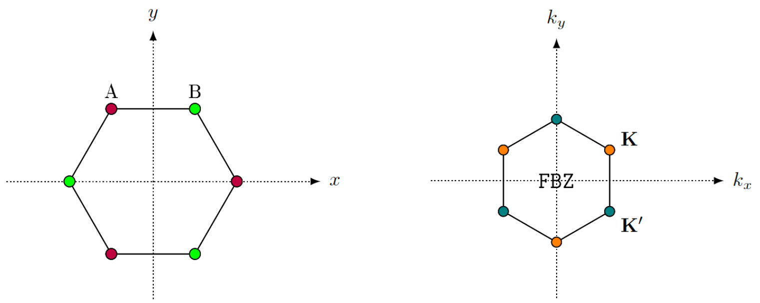
mass $\boldsymbol{m = \frac{3}{2}\,\tau}$ $\longleftarrow$ **torsion**

Killing spinor equation for the boundary supersymmetry

Solutions of the Dirac equation correspond to supersymmetries of the system.

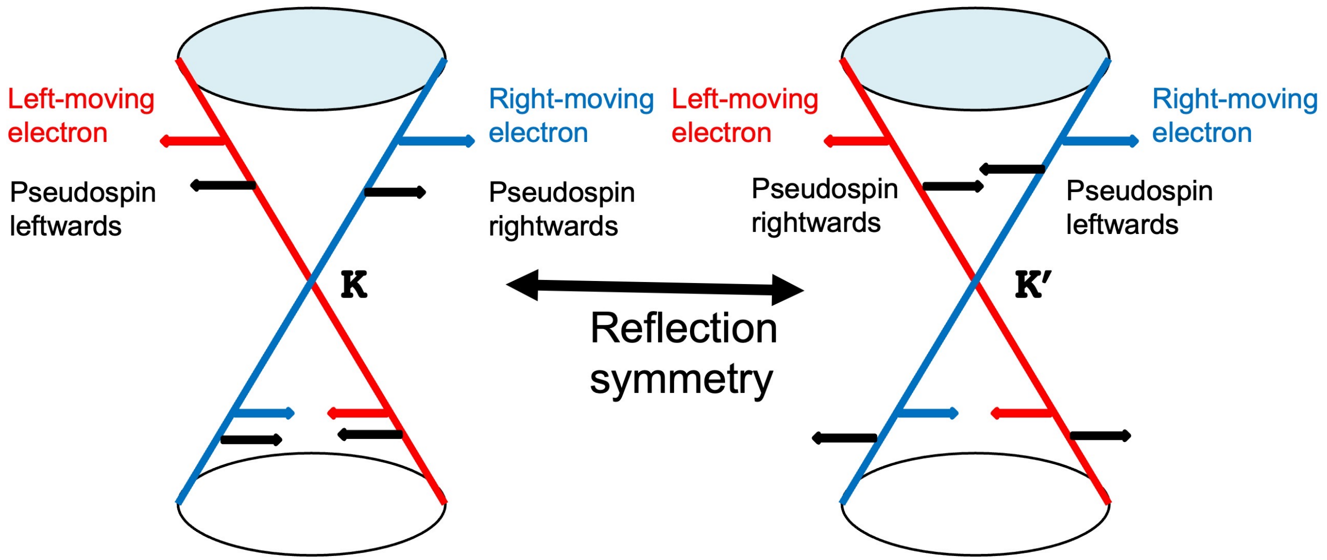
Electronic properties of the pseudoparticles, such as e.g. optical conductivity, are determined by supersymmetry.

Application to Graphene and the **K** and **K'** valleys



The reciprocal lattice of graphene is also a honeycomb lattice, rotated by an angle of $\pi/2$, featuring two inequivalent types of Dirac points: K and K'.

The corresponding Dirac equations are mapped to each other by a reflection symmetry



Mass terms in graphene like 2D materials

Mechanisms for opening mass gaps in graphene-like 2D materials include:

1) Breaking sublattices equivalence generating a parity odd mass term *M* (Semenoff model [G. W. Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)]), **e.g. by depositing a graphene monolayer on a suitable substrate, for instance of boron nitride or silicon carbide.**

2) Introducing a suitable periodic local magnetic flux (Haldane model [F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)]), **inducing an Aharonov-Bohm phase φ, breaking time-invariance:**

$$m_K = -3\sqrt{3}\, t_2 \sin\varphi; \qquad m_{K'} = 3\sqrt{3}\, t_2 \sin\varphi$$

with t_2 hopping amplitude between next-to-nearest neighbors.

Comparison with microscopic models of graphene-like 2D materials

More supersymmetry allows to describe the K and K' valleys in the first Brillouin zone of the reciprocal lattice.

In the special case $p = q$ a manifest parity symmetry emerges in the model, under the exchange of OSp(p|2) and OSp(q|2) in OSp(p|2)×OSp(q|2)

⇒ The ± sectors, being related by the reflection symmetry in one spatial axis, can be naturally identified with the K and K' valleys.

The masses generated by the torsion in the two sectors are:

$$\boxed{m_{\pm} = \frac{3}{2}\tau \mp 3\frac{f}{l},}$$

with τ parity odd, f even.

radius of Anti de Sitter space

Semenoff mass $M = \frac{3}{2}\tau$ **Haldane mass** $\sqrt{3}\, t_2 sin\, \varphi = \frac{f}{l}$

⇓

The Haldane and Semenoff-type masses are identified with geometric properties of the model, such as torsion/ curvature.

Outlook: Possible generalizations to other models

- **Different model for graphene like materials: In** [A. Iorio and P. Pais, Annals Phys. 398 (2018) 265–286, arXiv:1807.0876] **a different model is constructed, starting from a superalgebra SU(2|1, 1), whose bosonic subgroup contains SU(1,1) x SU(2): suitable to describe topological features of graphene such as grain boundaries. In our construction we can reproduce a similar case by starting from $\mathcal{N}$=4 and choosing p=4, q=0, with supergroup OSp(4|2) x SO(2,1), which has bosonic subgroup SO(4) x SO(2,1)=SU(2) x SU(2) x SO(2,1).**

- **Addition of spin: More supersymmetry, e.g. such as a generalization to the exceptional supergroup D(2,1;α)** [L. Andrianopoli, BLC, R. Matrecano, R. Noris, L. Ravera, M. Trigiante, *Twisting D(2,1; α) Superspace,* Fortsch.Phys. 69 (2021) 10, 2100111, arXiv:2107.10361]**, allows the addition of a non-Abelian gauge field and to study of the spin-orbit interaction and the quantum spin Hall effect, first postulated in graphene in** [C.L. Kane, E.J. Mele, PRL 95 (22), 226081, arXiv:cond-mat/0411737], **but more easily testable in small gap semiconductors like Hg Te/Cd Te (mercury-, cadmium- telluride)** [M. König et al., Science Express Research Articles. 318 (5851): 766770, arXiv:0710.0582 [cond-mat.mes-hall]] **with strong spin-orbit coupling.**

- **More general Weyl semimetals, topologically non trivial materials in higher dimensions?**

Outlook: Holography and topology

- **Holographic renormalization: With O.Miškovic and R. Olea** [L. Andrianopoli, BLC, R. Matrecano, O. Mišković, R. Noris, R. Olea, L. Ravera, M. Trigiante, *$\mathcal{N}$ = 2 AdS_4 supergravity, holography and Ward identities*, JHEP 02 (2021) 141, arXiv:2010.02119], **we are explictly computing the holographic map, in order to apply the holographic renormalization scheme to our AdS_4/graphene correspondence:**

  the counterterms should sum up to topological invariants [R.Aros, M.Contreras, R.Olea, R.Troncoso, J.Zanelli, Phys.Rev.Lett. 84 (2000) 1647-1650, arXiv:gr-qc/9909015].

- **BRST quantization** [L. Andrianopoli, B.L. Cerchiai, P.A. Grassi, M. Trigiante, The Quantum Theory of Chern-Simons Supergravity, JHEP 1906 (2019) 036, arXiv:1903.04431]: **The identification of the graphene worldvolume Lorentz group with the supergravity target space-time symmetries defines a topological twist, along the lines of** [Kapustin, Saulina, Nucl.Phys. B 823 (2009) 403]: **Chern-Simons theory on a SuperGroup with a gauge fixing of fermion gauge symmetries ≅ topologically twisted super-Chern-Simons theory coupled to SUSY matter fields. It corresponds to a topologically inequivalent corner of the theory and paves the way to the investigation of the embedding of the model in string theory** [Gaiotto, Witten, JHEP 1006 (2010) 097] **and its understanding within the rich web of dualities existing in 2+1 dimensions.**

Conclusions

- In the framework of holography, we have obtained a description of 2D graphene-like materials in a suitable AdS_3 patch at the boundary of an extended supergravity in one dimension higher with $\mathcal{N}$ supersymmetries.

- The model features supersymmetry, and it can be viewed as a top-down approach to understand the origin of the observed supersymmetric phenomenology in graphene [S.-S. Lee, "Emergence of supersymmetry at a critical point of a lattice model", Phys. Rev. B76 (2007) 075103, cond-mat/0611658].

$\downarrow$

- This top-down approach is more predictive than the common bottom-up one, because it is strongly constrained from the supersymmetry of the underlying gravity theory. Solutions are in one-to-one correspondence with supersymmetries of the system.

$\downarrow$

- In collaboration with J. Zanelli we are explicitly constructing and studying some solutions [L. Andrianopoli, BLC, R. D'Auria, A. Gallerati, R. Noris, L. Ravera, M. Trigiante, J. Zanelli, in progress]

- Topological properties of the D = 2 +1 theory: In collaboration with R. Olea and J. Zanelli we are studying the topological properties of the theory in D=2+1, particularly at the boundary of a 1+1 interface with the aim of characterizing boundary currents in the presence of domain walls.

Thank you!