Criticality induced compatibility in multiparameter metrology

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Introduction

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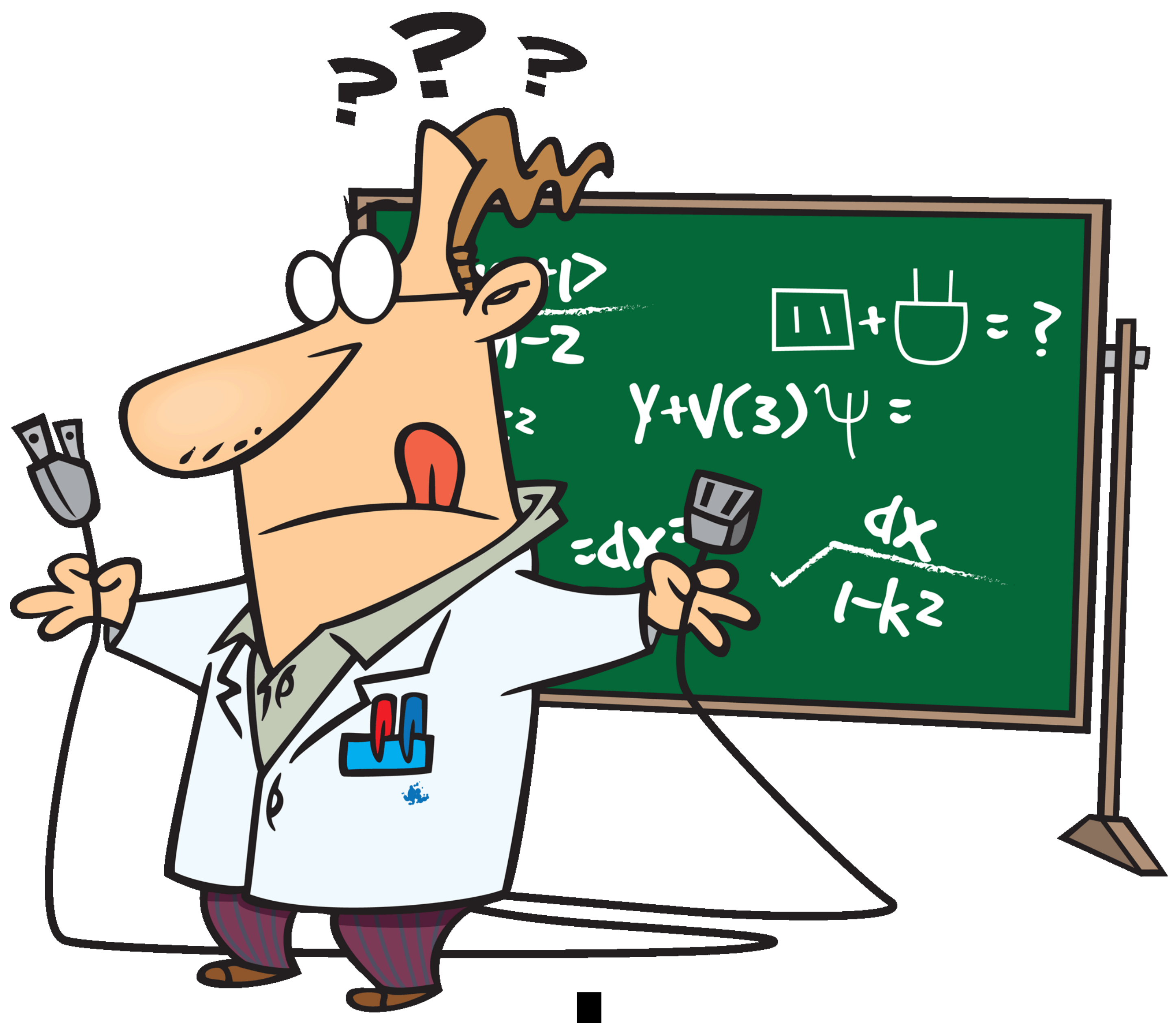
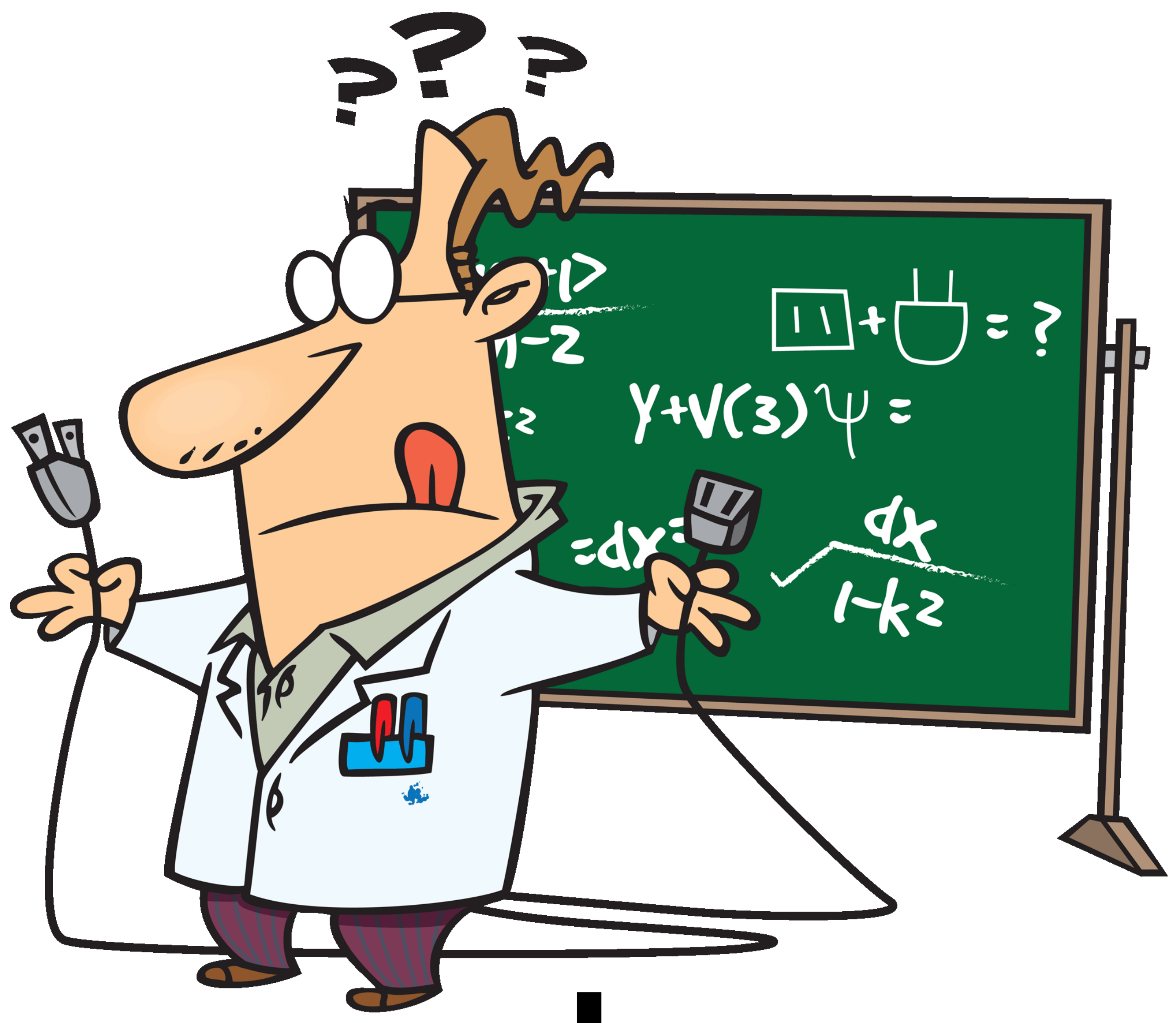


$$\bar{\theta}$$

Introduction

$$\left(\theta_0, \theta_1, \ldots, \theta_N\right)$$



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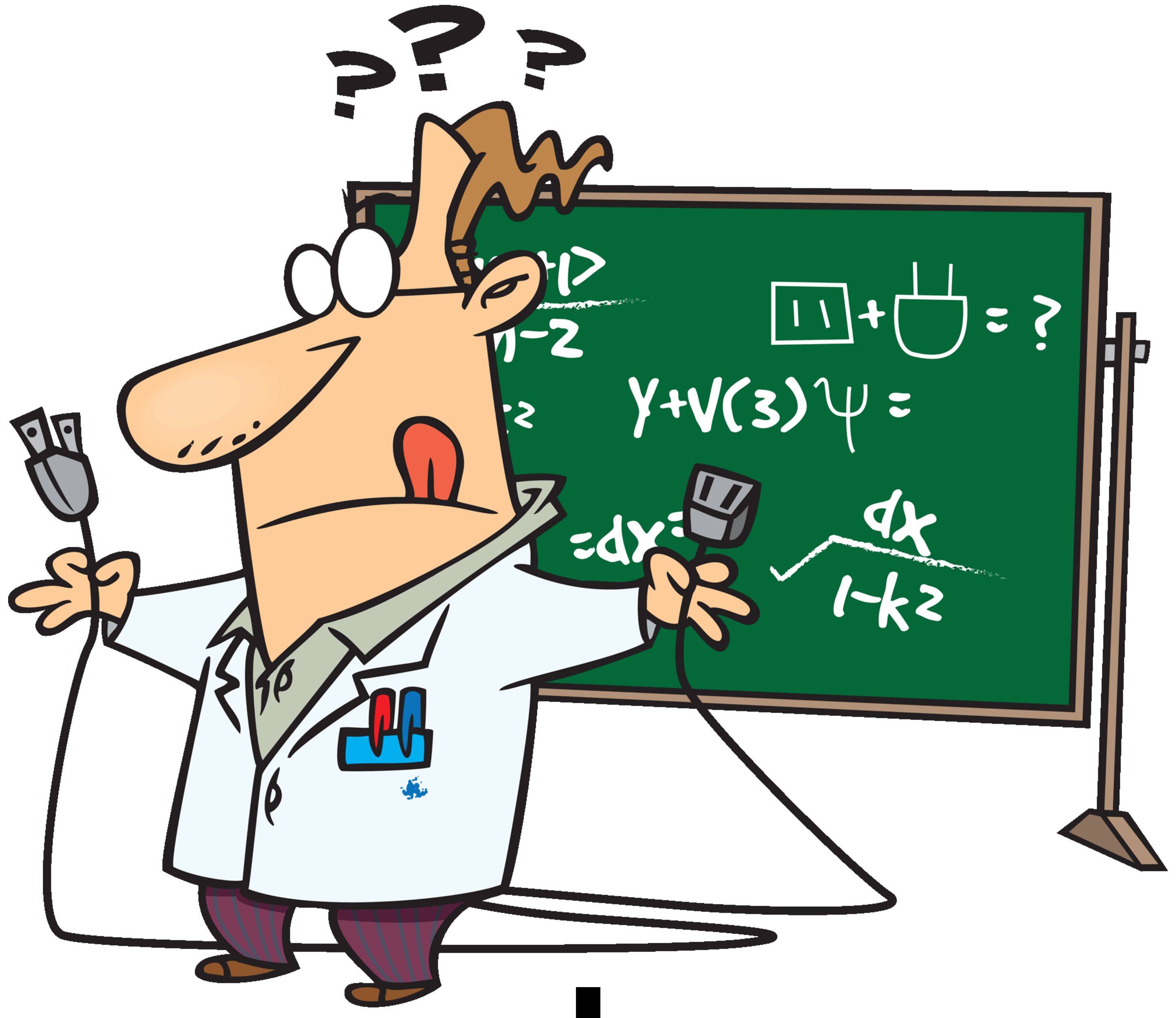

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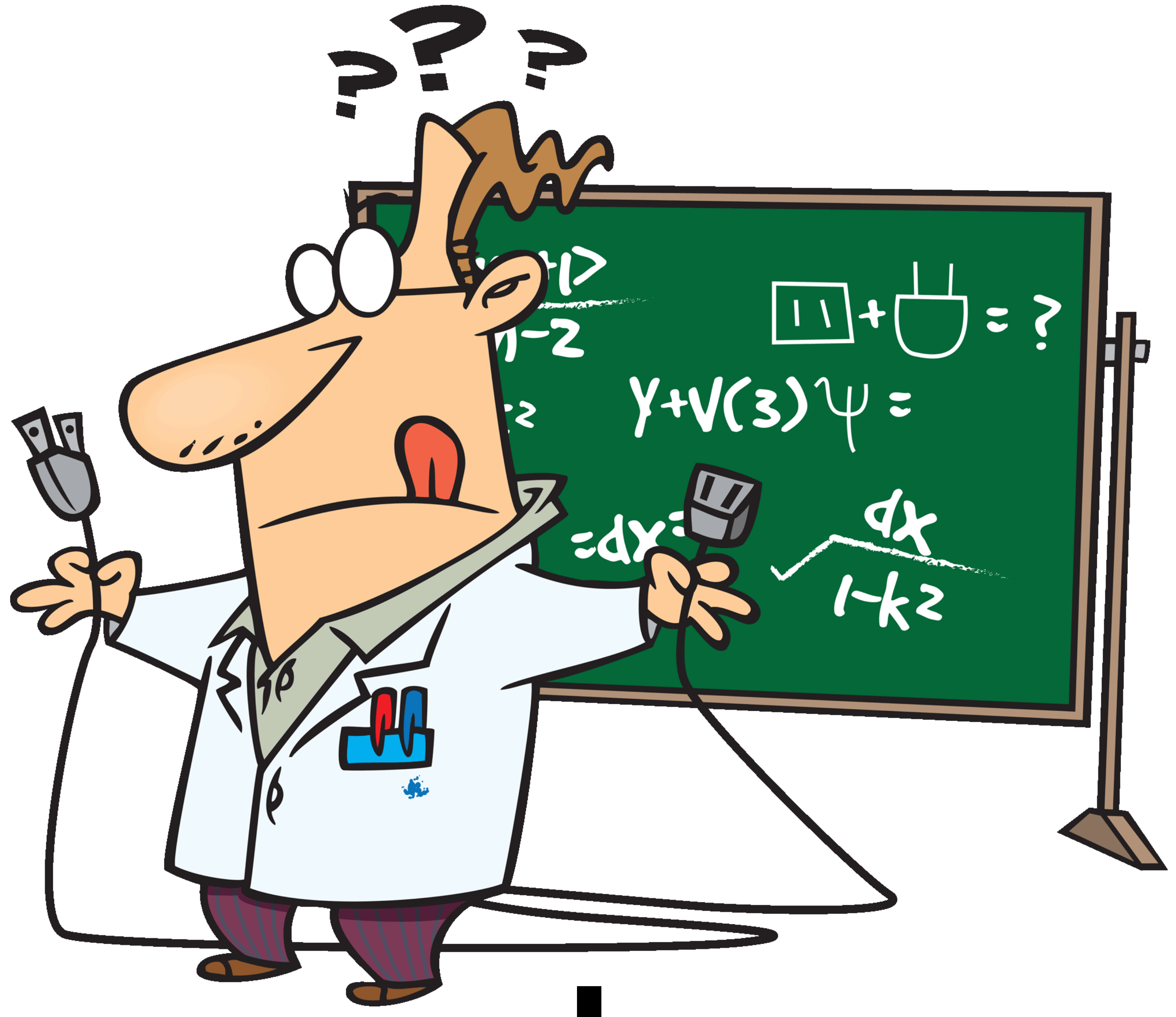

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Steven M. Kay. Fundamentals of Statistical Signal Processing: Estimation Theory. USA: Prentice-Hall, Inc., 1993.

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↑

Quantum Fisher Information

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Quantum Fisher Information

$$\frac{\partial \rho_\theta}{\partial \theta_i} = \frac{1}{2}\left\{L_{\theta_i}, \rho_\theta\right\}$$

Symmetric logarithmic derivative

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W weight matrix

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$$0 \leq D\,(W) \leq \mathrm{tr}\left(WF_Q^{-1}\right) R_\lambda$$ $\longrightarrow$ **Quantumness**

Angelo Carollo et al. "On quantumness in multi-parameter quantum estimation", Journal of Statistical Mechanics: Theory and Experiment 2019

Quantumness

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Two parameter case

$$R_\lambda^{(2)} = \sqrt{\frac{\det(2U)}{\det\left(F_Q\right)}}$$

Quantum phase transition

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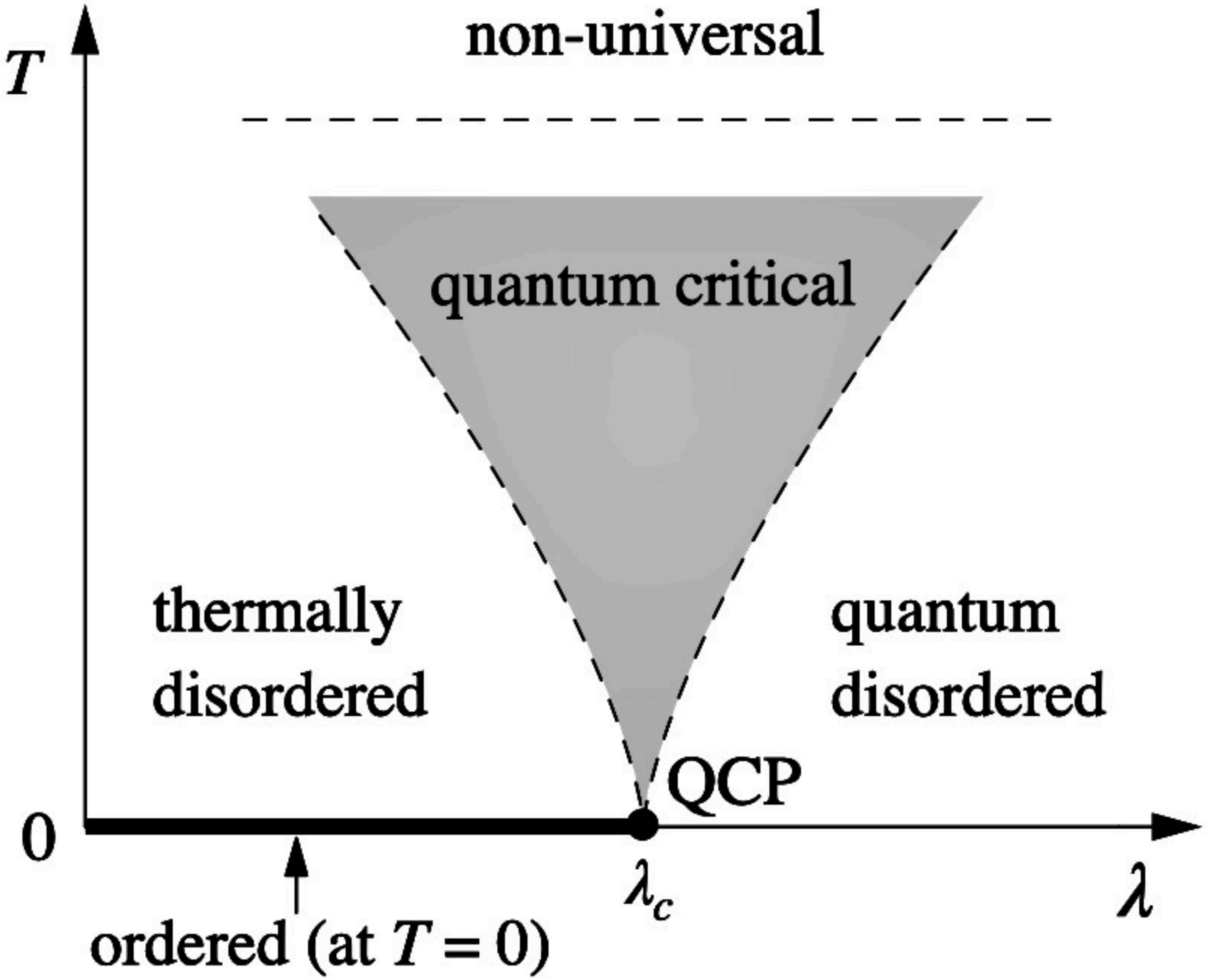

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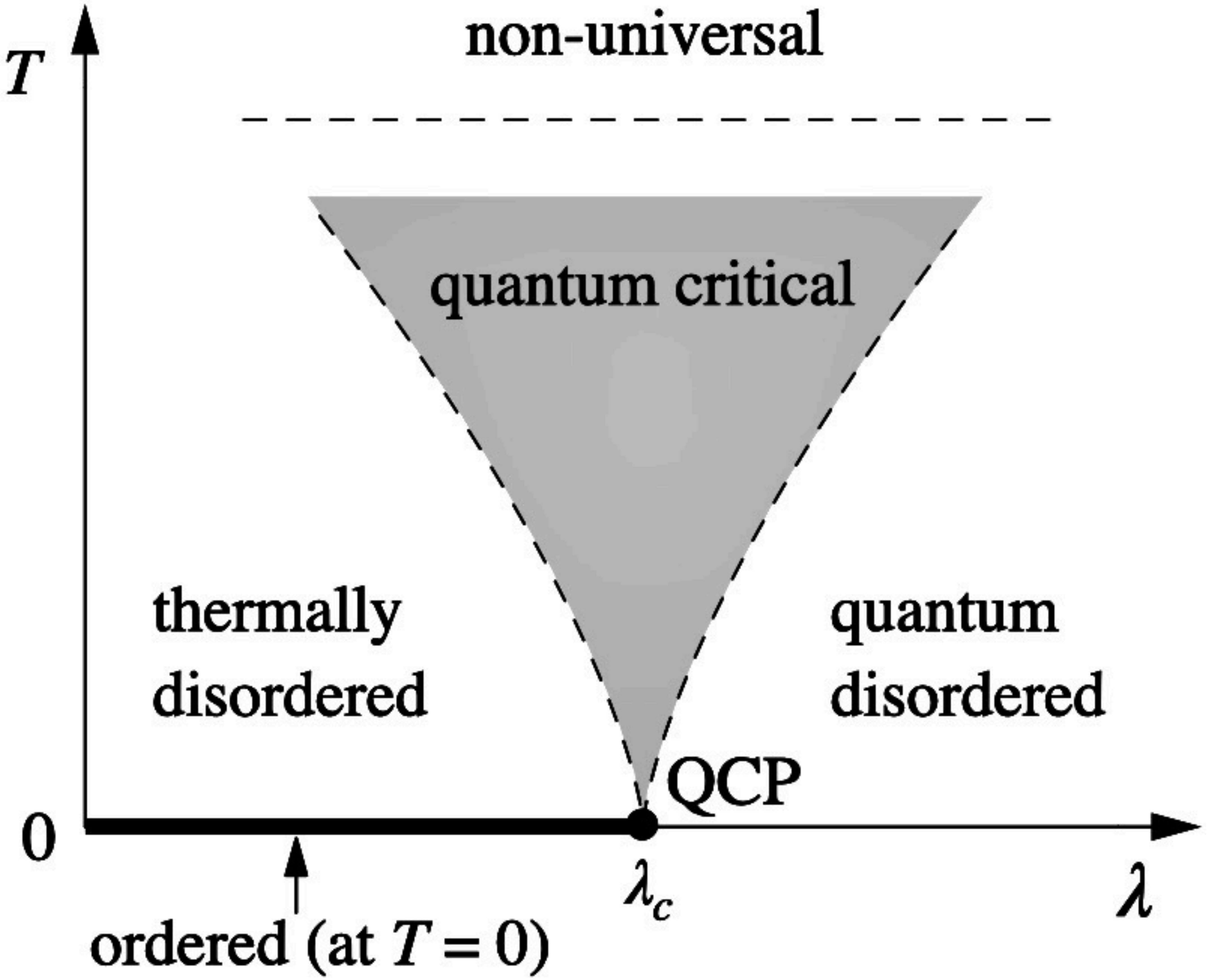

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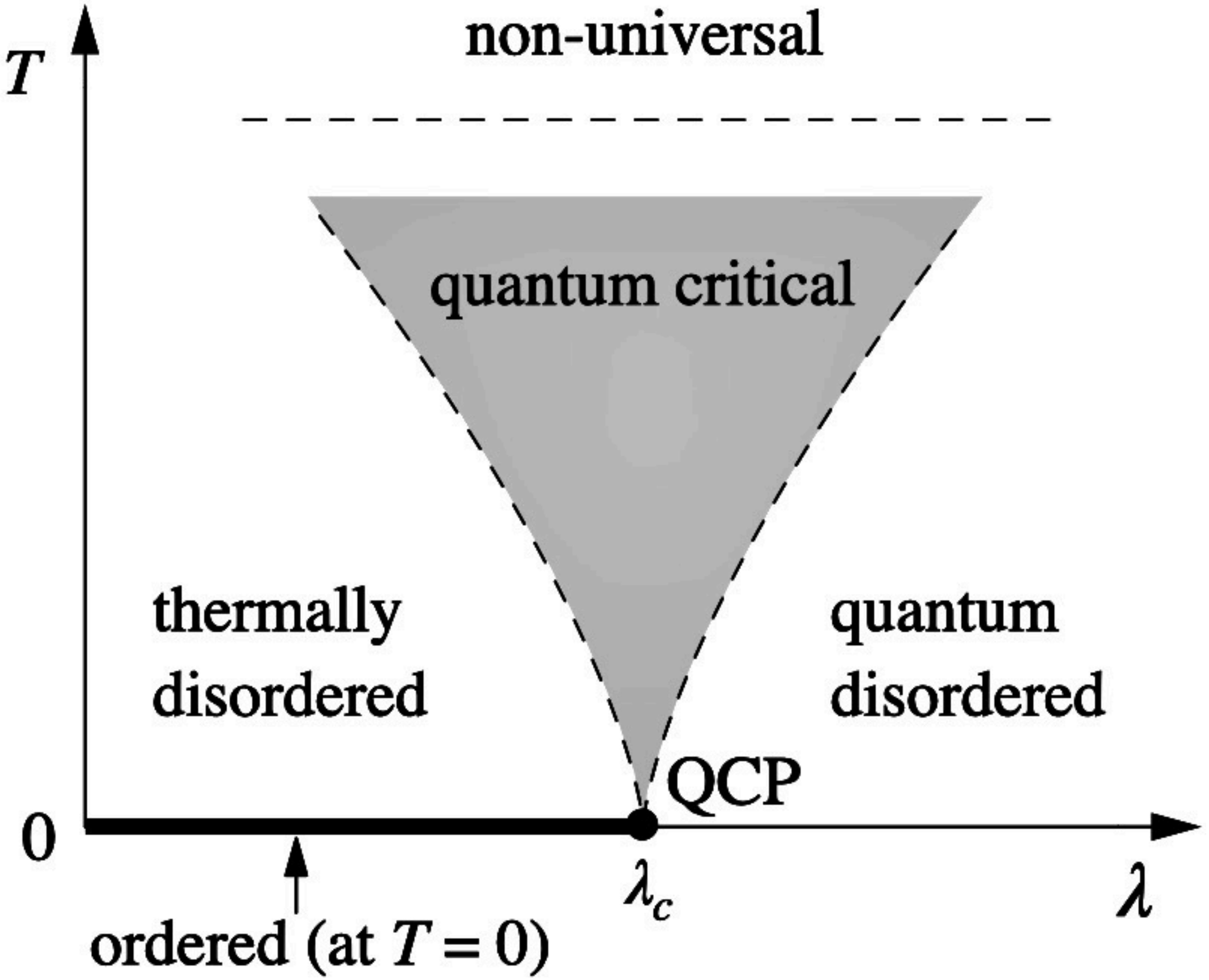

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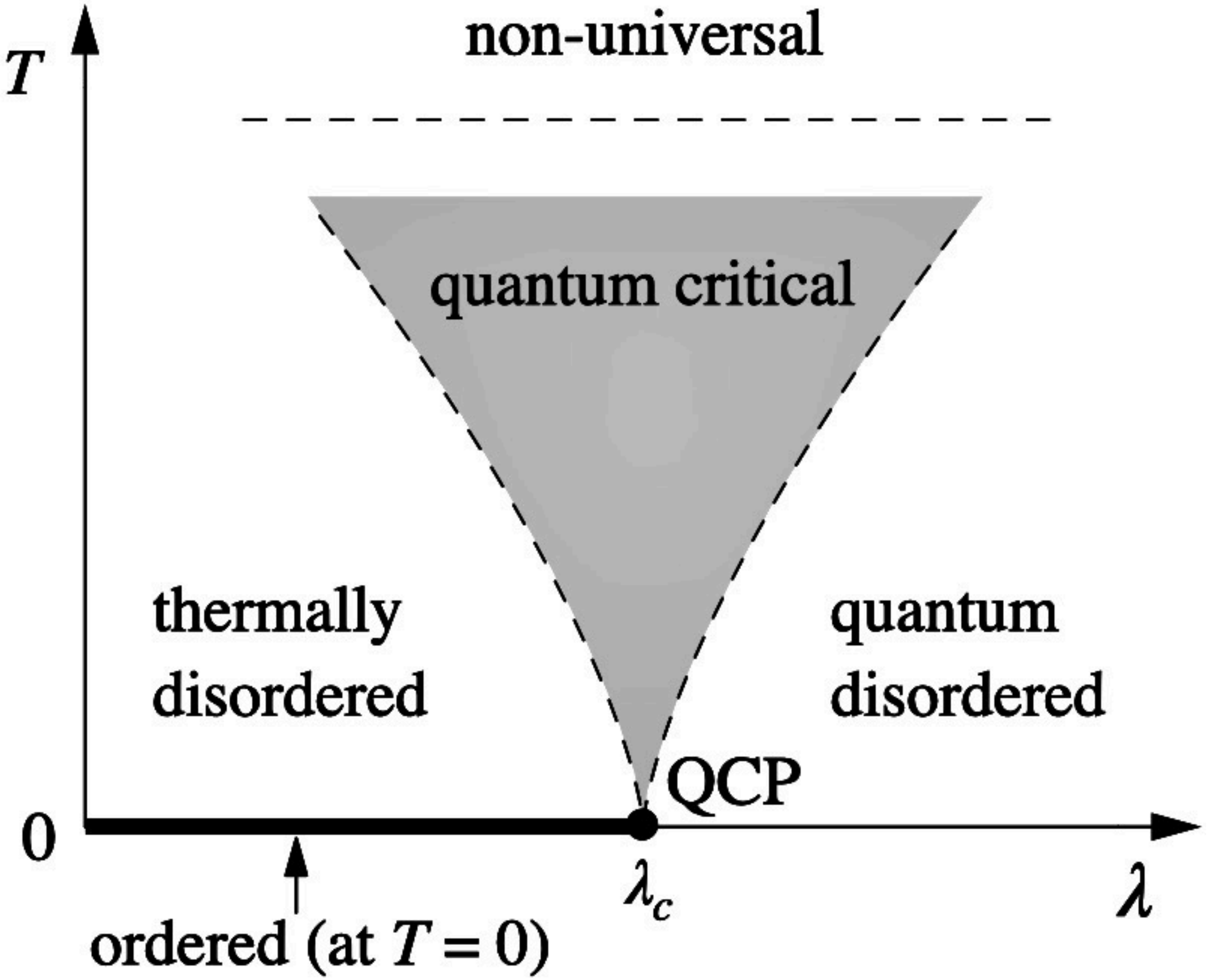

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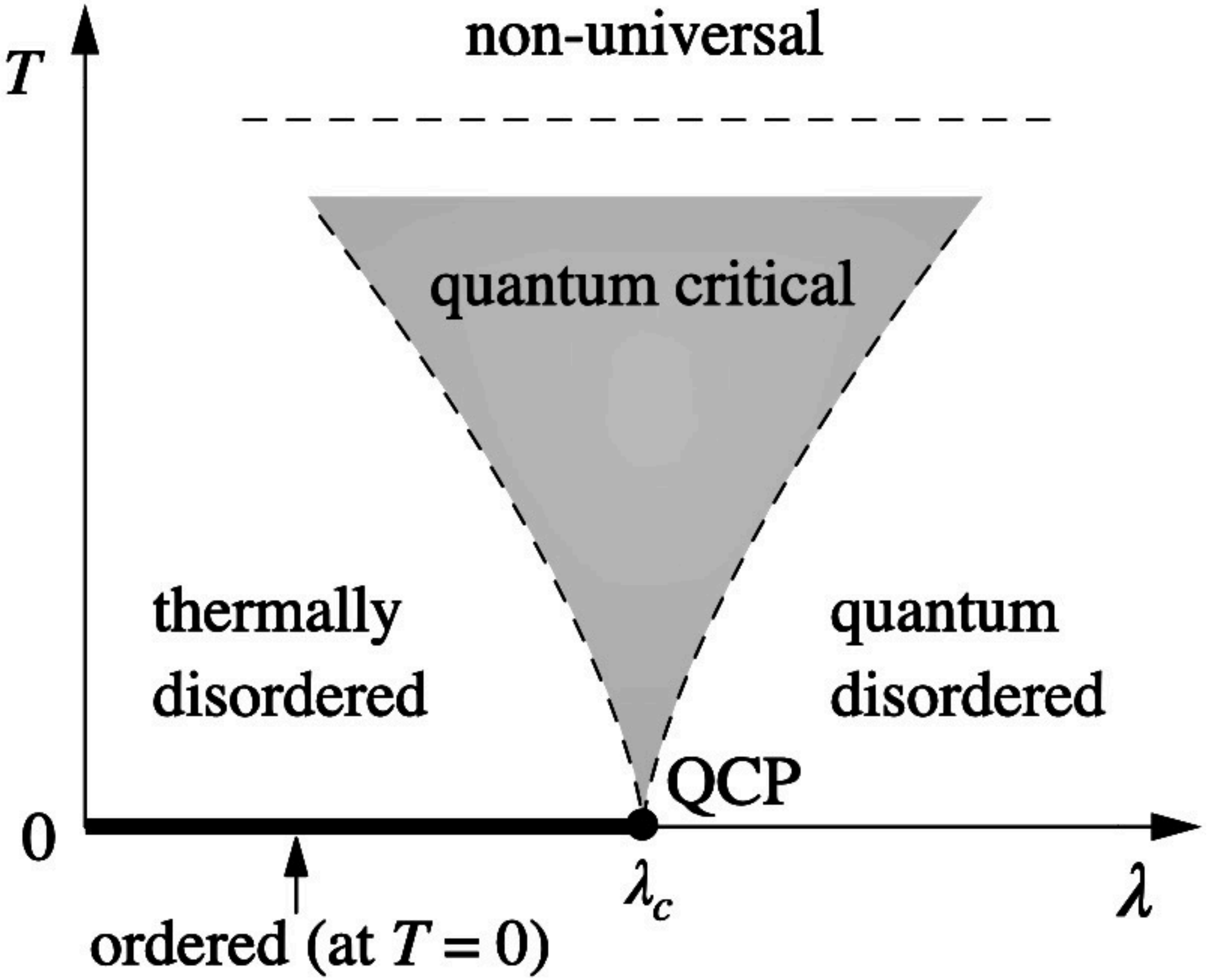

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$$R \sim \frac{y_U \Delta_0^2(L)}{(\partial_\mu E_\mu)(\partial_\nu E_\nu)} \leq 1$$

Systems

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Ferromagnetic $\longrightarrow$ $$H = -\sum_i \sigma_i^z \sigma_{i+1}^z + h_x \sigma_i^x + h_y \sigma_i^y + h_z \sigma_i^z$$

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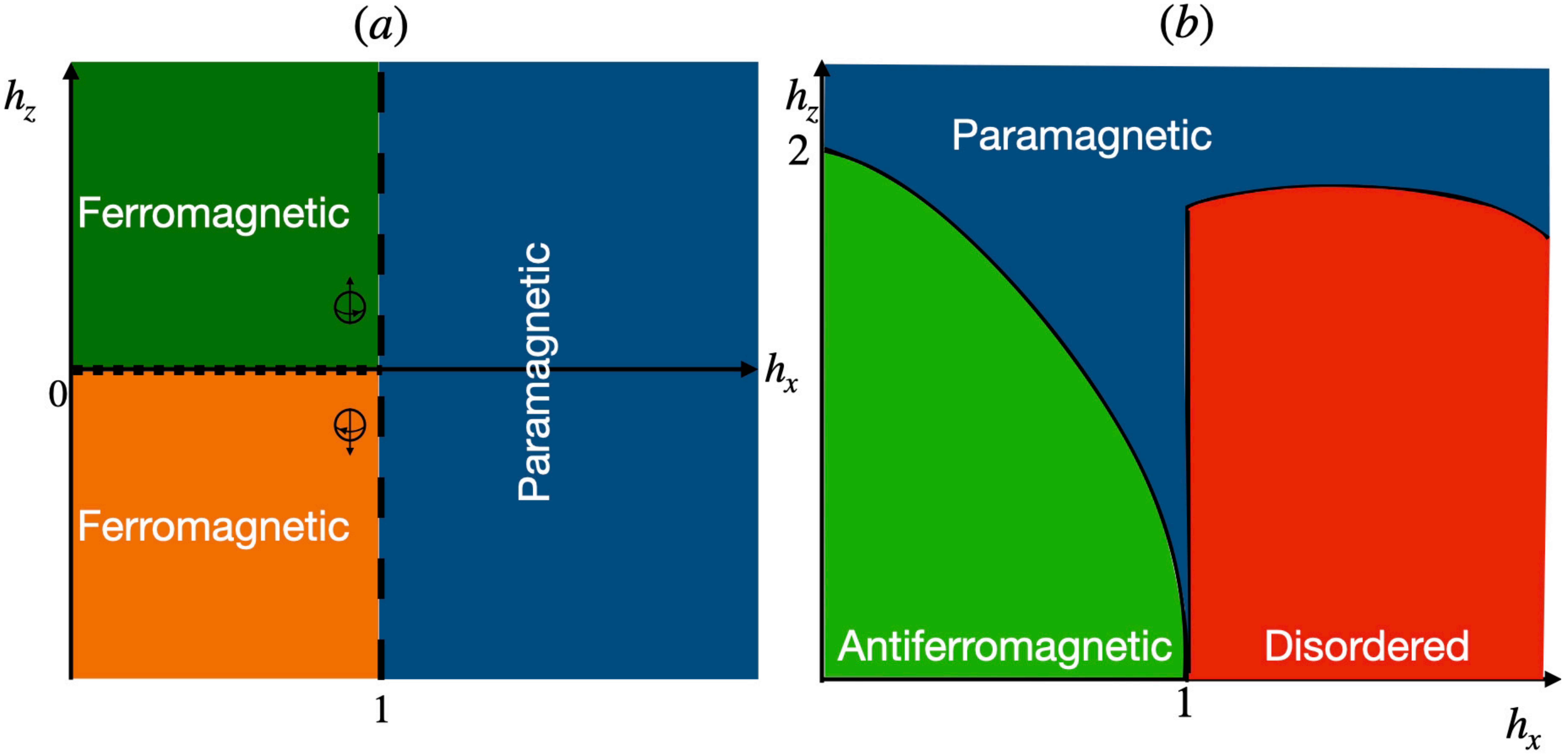
Phase diagrams



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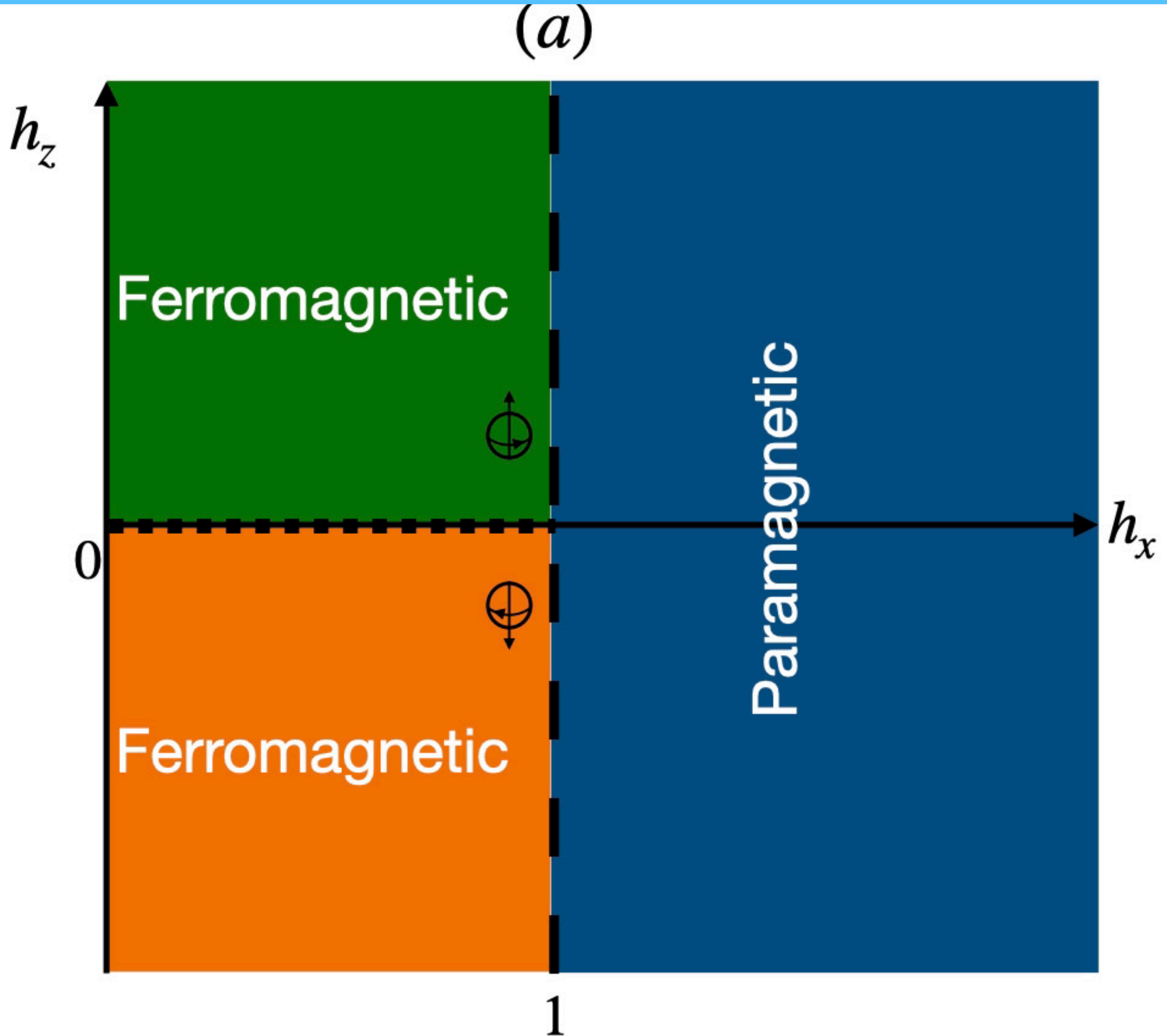
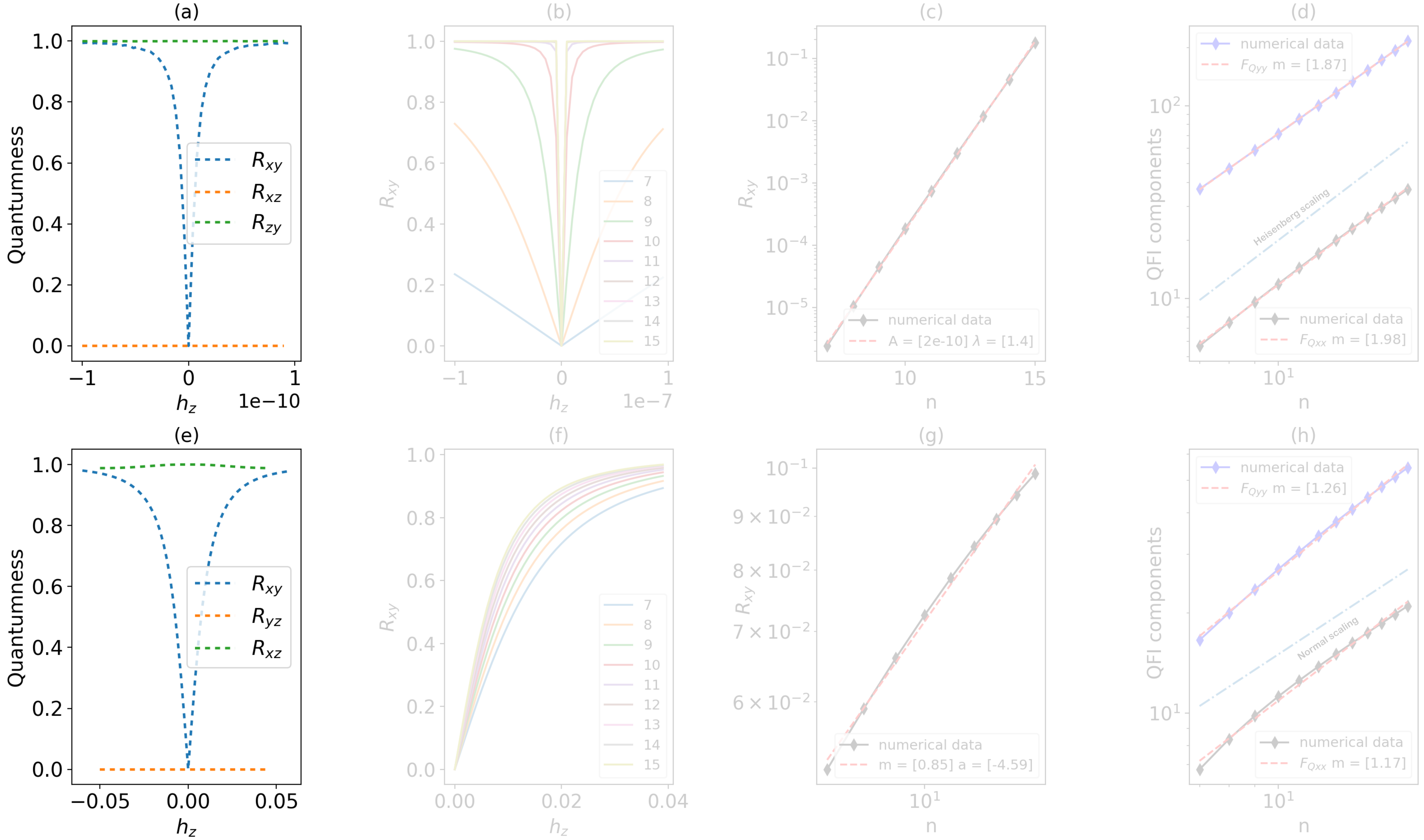


Ferromagnetic Results



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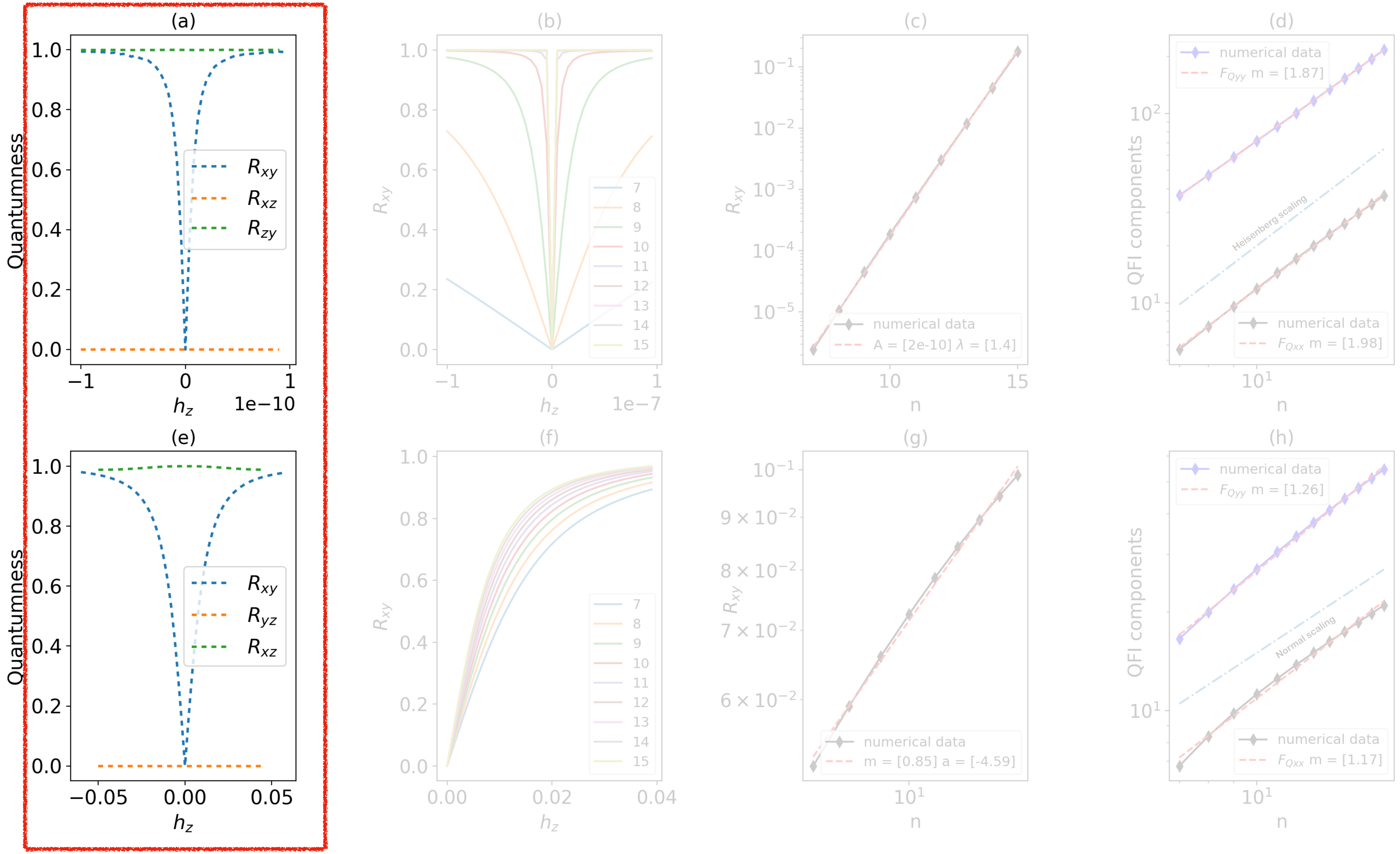

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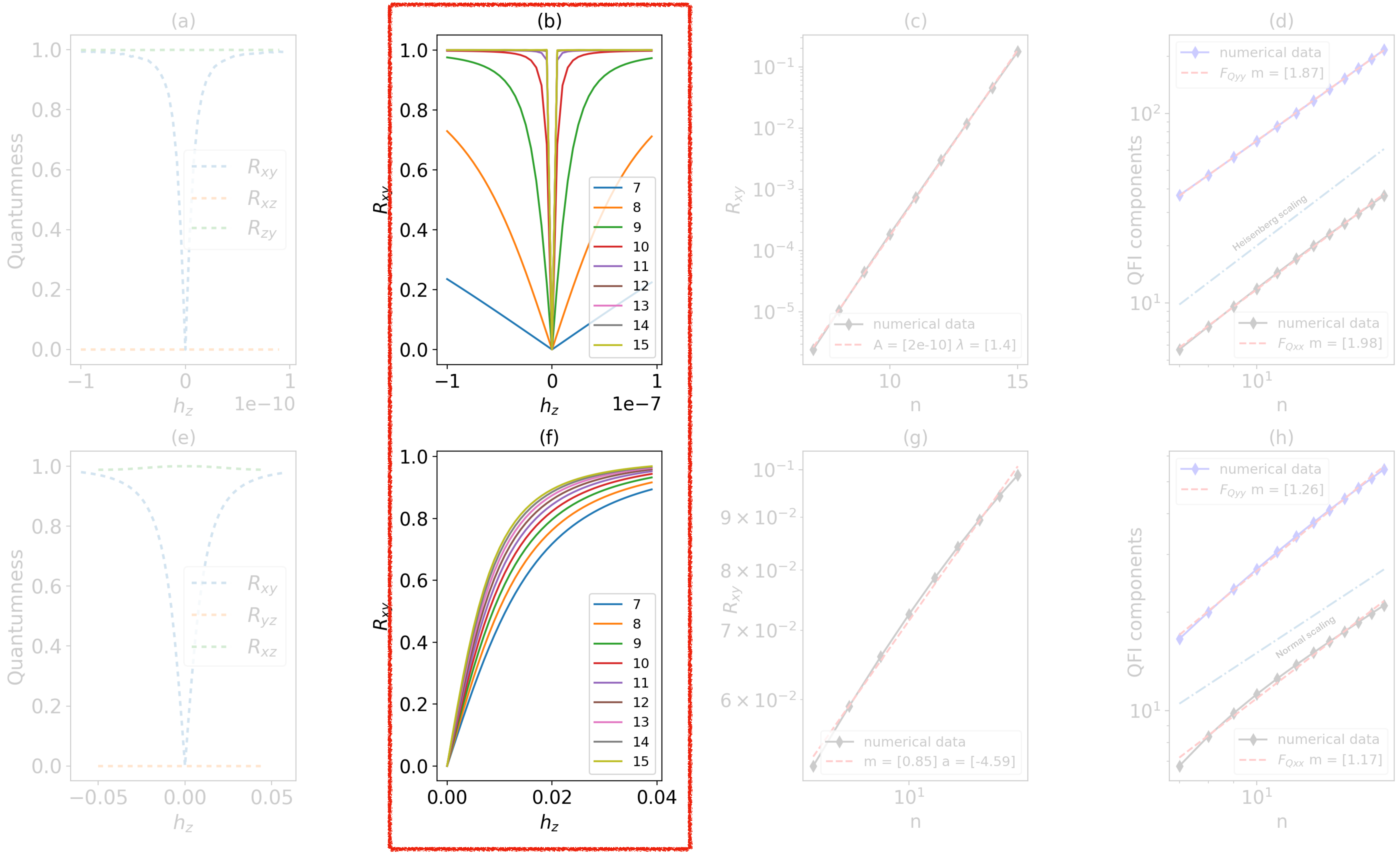

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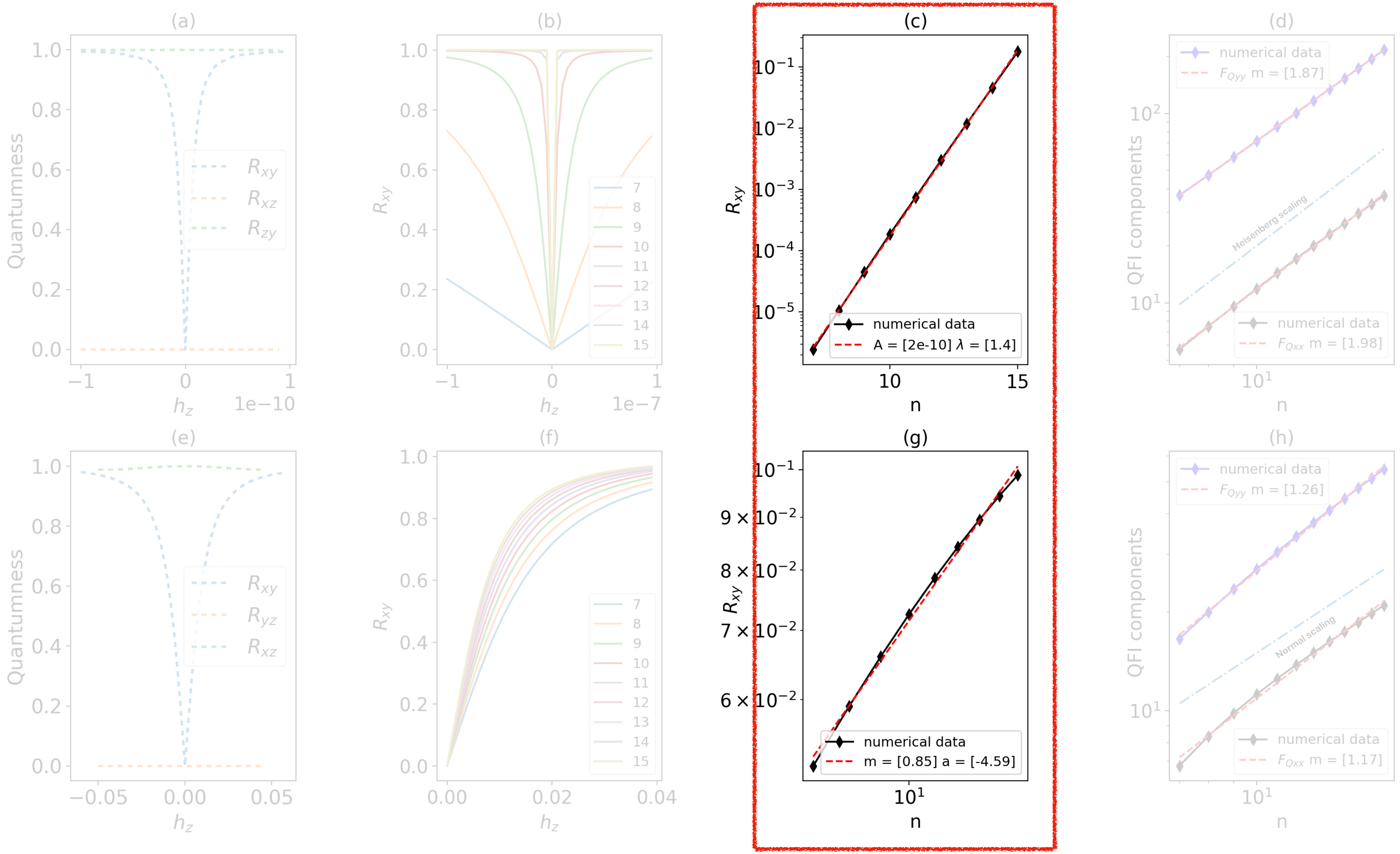

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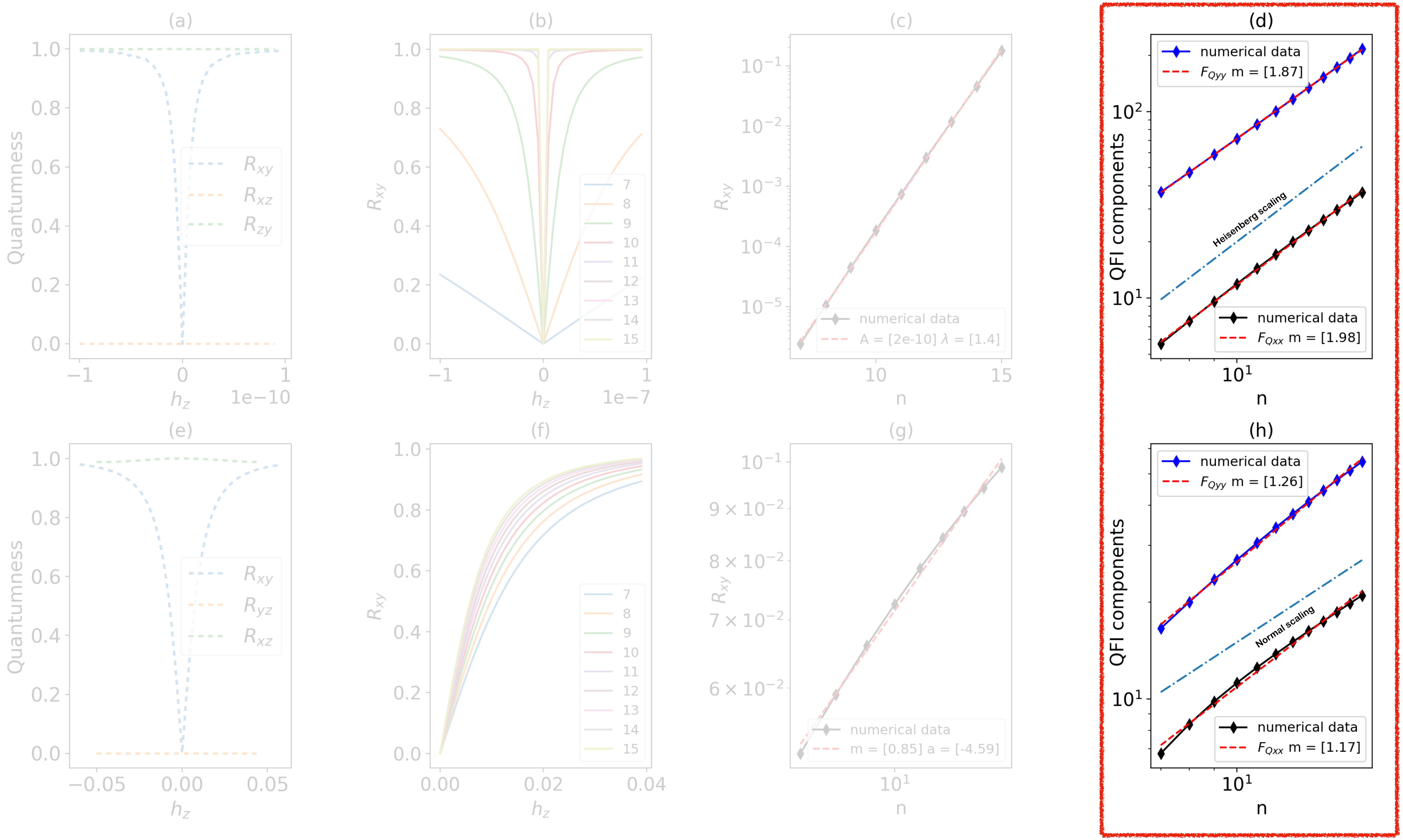

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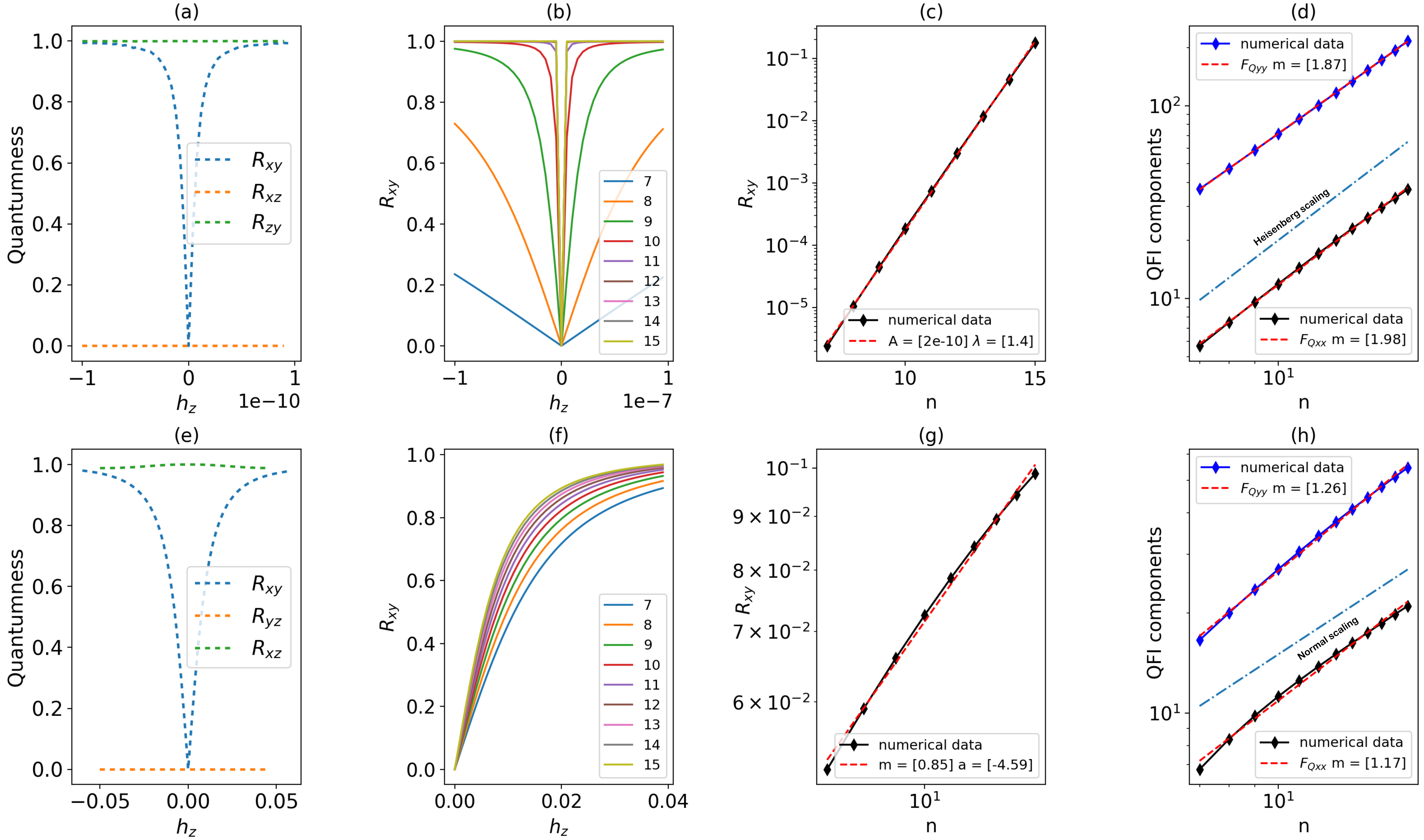

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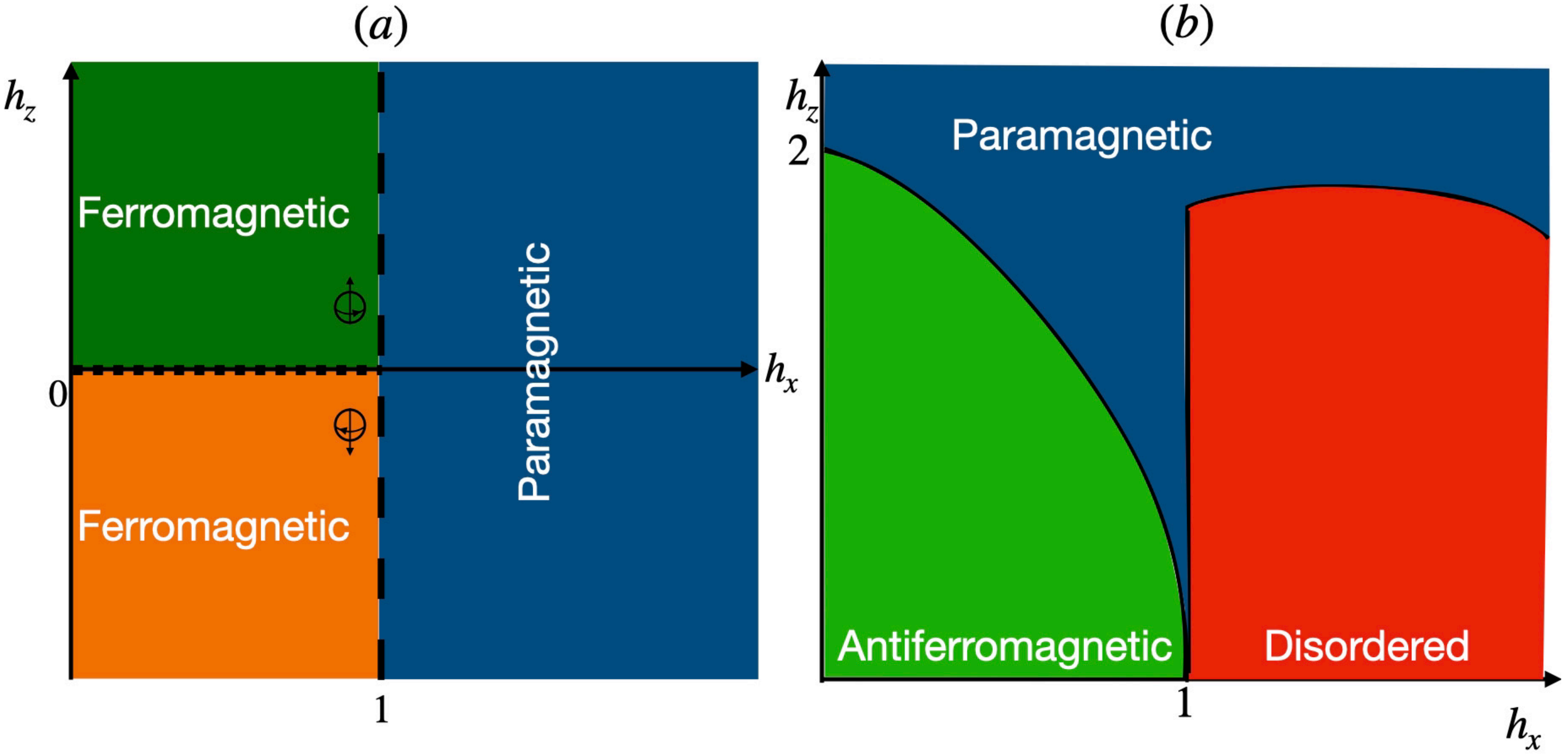

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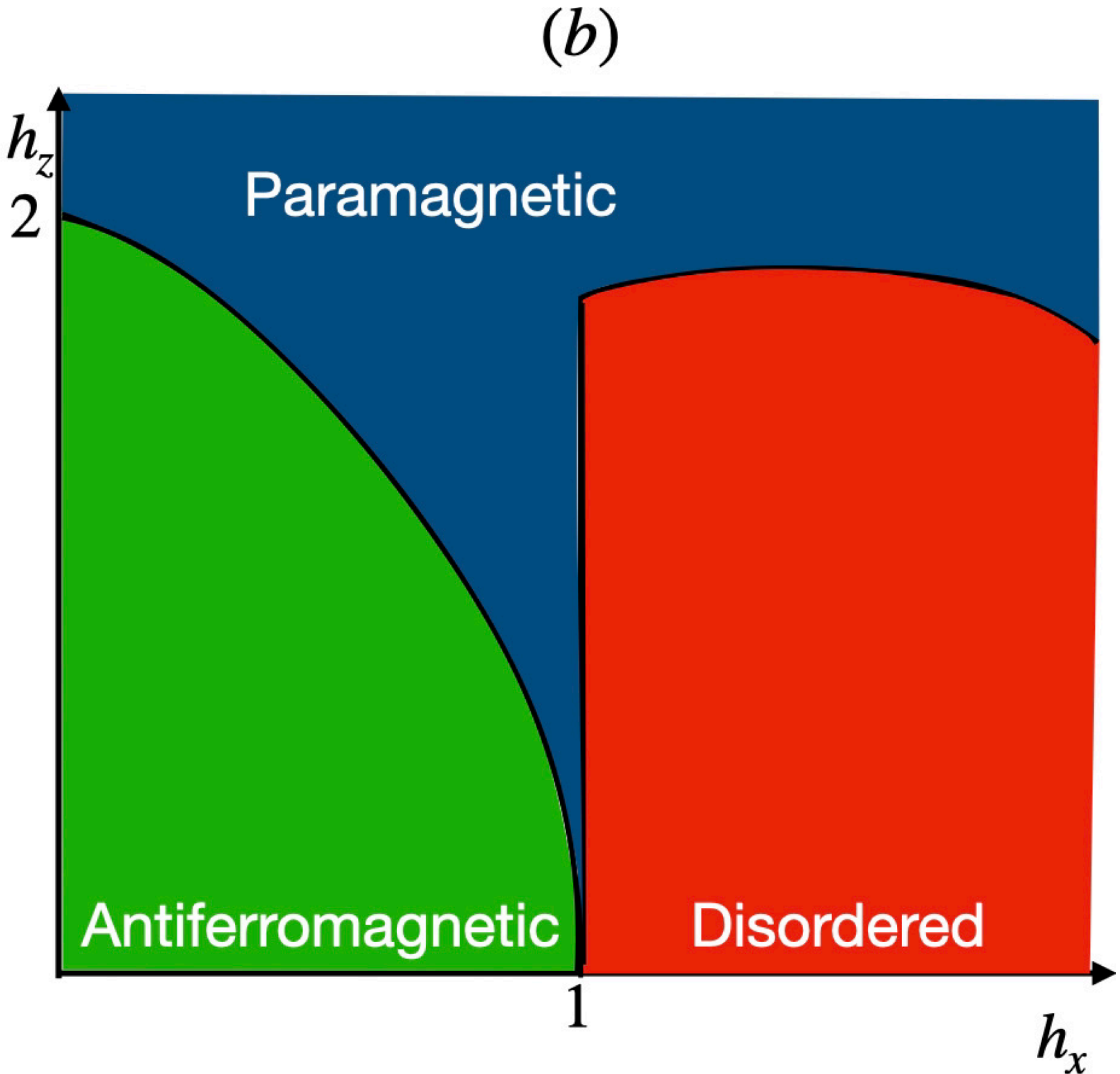

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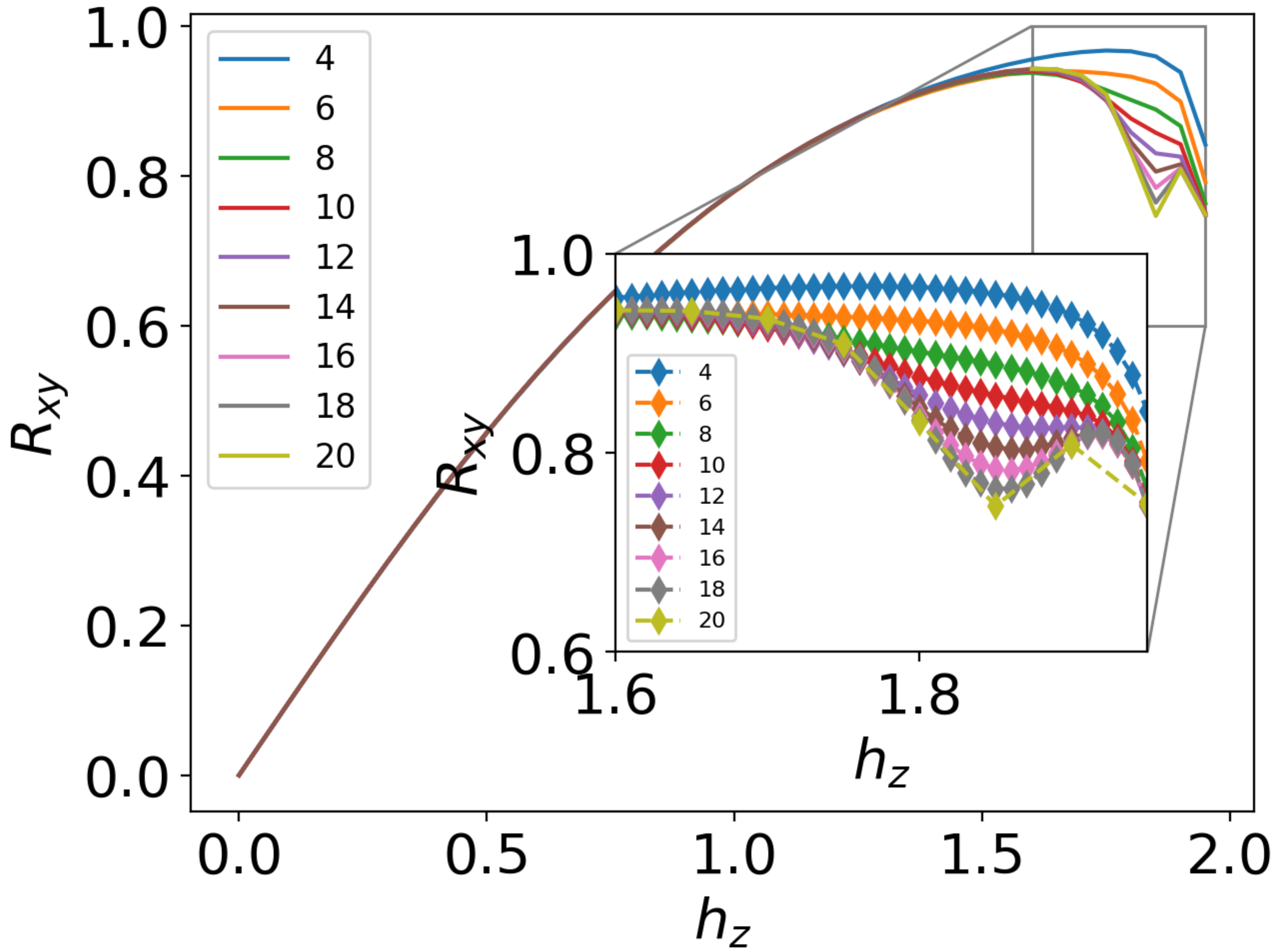

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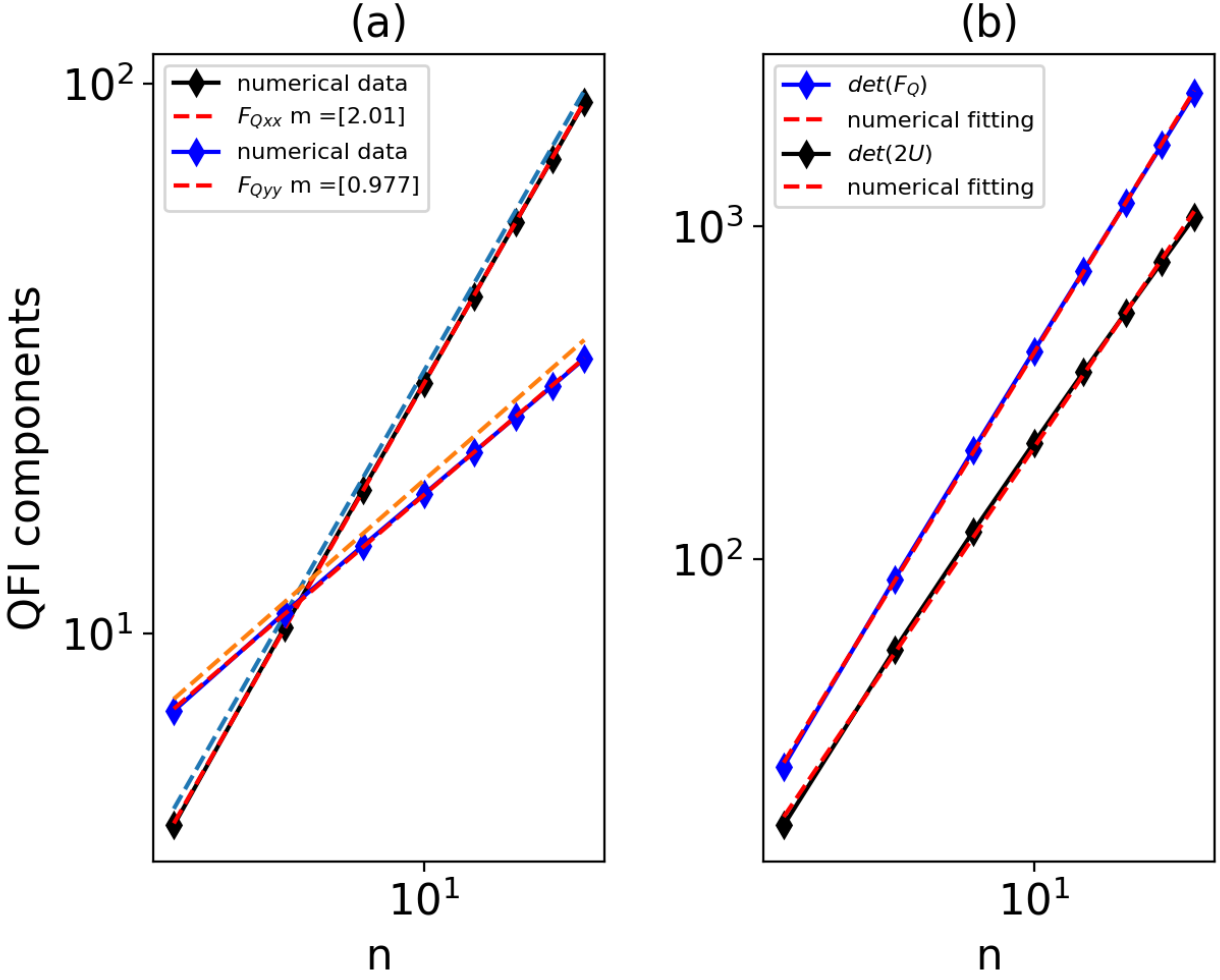

Conclusion and outlooks

Main results:

Scaling analysis of the quantumness

Variegated dependence of the quantumness from criticality in representative systems.

- Continuous phase transition
- First order phase transition

Outlooks:

- Quantumness in out of equilibrium conditions (quenches)
- Estimation in presence of noise
- MPS approach to many-body quantum metrology

G. Di Fresco, B. Spagnolo, D. Valenti, and A. Carollo, “Multiparameter quantum critical metrology,” arXiv 2203.12676, 2022. SciPost physics: In press